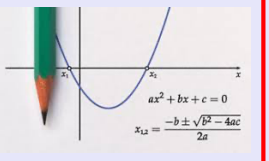
Math 125

Spring 2022

Lecture 27



Class QZ 20

Solve $(\sqrt{x}-1)^2-5(\sqrt{x}-1)+6=0$

Let $u=\sqrt{x}-1$

$u^2-5u+6=0$

$(u-2)(u-3)=0$

$u=2$ $\quad$ $u=3$

when $u=2$: $\sqrt{x}-1=2$, $\sqrt{x}=3$, $x=9$ ✓

when $u=3$: $\sqrt{x}-1=3$, $\sqrt{x}=4$, $x=16$ ✓

$\{9, 16\}$

Consider $(x+4)^2 + (y-3)^2 = 25$

Circle $(x-h)^2 + (y-k)^2 = r^2$

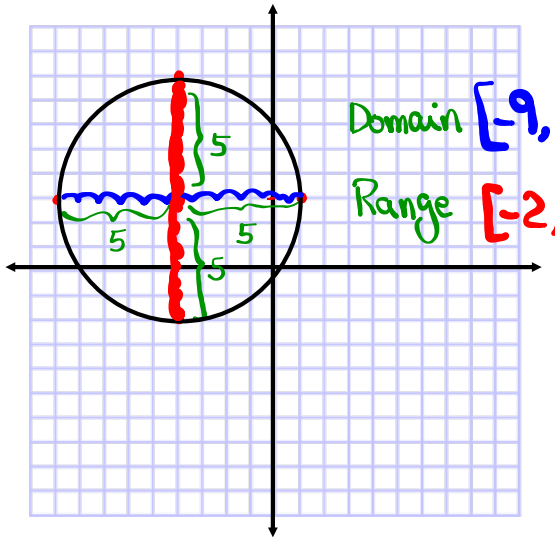
$h = -4$

$k = 3$

$r^2 = 25$ $r = 5$

Center $(h,k) = (-4,3)$

Radius $r = 5$



Domain $[-9,1]$

Range $[-2,8]$

Consider $\frac{(x-5)^2}{9} + \frac{(y+4)^2}{25} = 1$

Ellipse $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

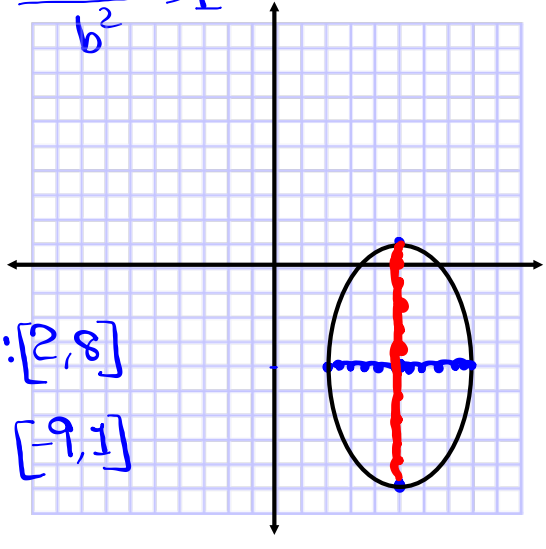
$h = 5$

$k = -4$

Center $(h,k) = (5,-4)$

$a^2 = 9$ $a = 3$

$b^2 = 25$ $b = 5$



Domain: $[2,8]$

Range: $[-9,1]$

Hyperbola

1) opens Right & left

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

2) opens up & down

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Center (h, k)

From the center $\pm a$ units to right and left

From the center $\pm b$ units up and down

we draw the fundamental rectangle.

we draw the diagonals and extend them

when opens right and left
go to the midpoint of vertical sides of the rectangle, draw the branches within the diagonals.

when opens up and down
go to the midpoint of horizontal sides of the rectangle, draw the branches within the diagonals.

Ex: $\frac{x^2}{16} - \frac{y^2}{9} = 1$ → $(x-0)^2$ → $(y-0)^2$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

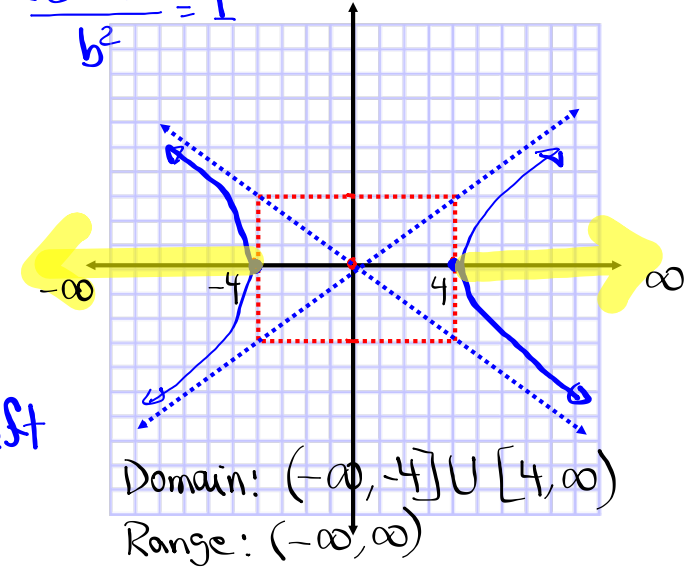
$h=0$, $k=0$

Center $(0,0)$

$a^2=16$ $a=4$

$b^2=9$ $b=3$

Opens right & left



Domain: $(-\infty, -4] \cup [4, \infty)$

Range: $(-\infty, \infty)$

Ex: $\frac{(y-2)^2}{4} - \frac{x^2}{9} = 1$

$\frac{(y-2)^2}{4} - \frac{(x-0)^2}{9} = 1$

$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$

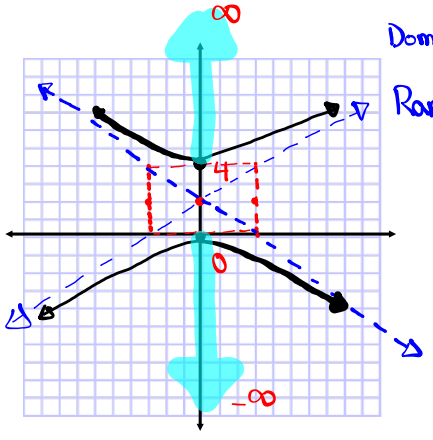
$h = 0$

$k = 2$ → Center $(0,2)$

$a^2 = 9$ $a = 3$

$b^2 = 4$ $b = 2$

opens up & down



Domain: $(-\infty, \infty)$

Range: $(-\infty, 0] \cup [4, \infty)$

Ex: $\frac{(x-2)^2}{4} - \frac{(y-4)^2}{16} = 1$

opens right & left

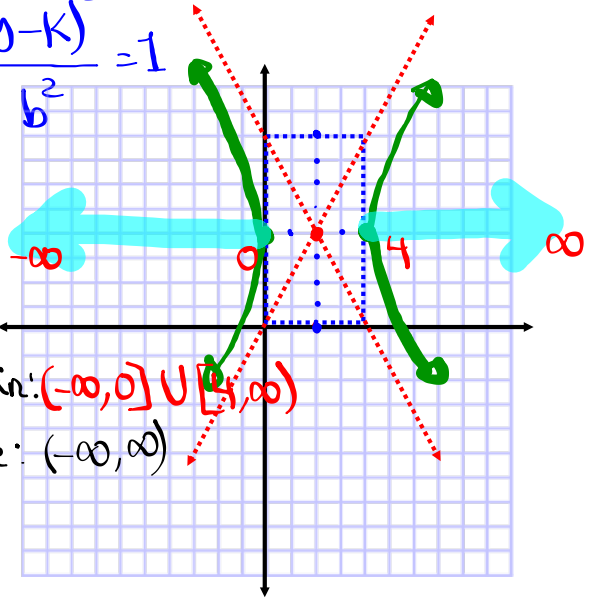
$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$

$h = 2$

$k = 4$ → Center $(2,4)$

$a^2 = 4$ $a = 2$

$b^2 = 16$ $b = 4$



Domain: $(-\infty, 0] \cup [4, \infty)$

Range: $(-\infty, \infty)$

Ex: $4(y+3)^2 - 25(x+4)^2 = 100$

We like the RHS to be 1,

So divide by 100

$$\frac{4(y+3)^2}{100} - \frac{25(x+4)^2}{100} = \frac{100}{100}$$

Simplify

$$\frac{(y+3)^2}{25} - \frac{(x+4)^2}{4} = 1$$

$$\frac{(y-k)^2}{b^2} - \frac{(x-h)^2}{a^2} = 1$$

Center $(-4,-3)$

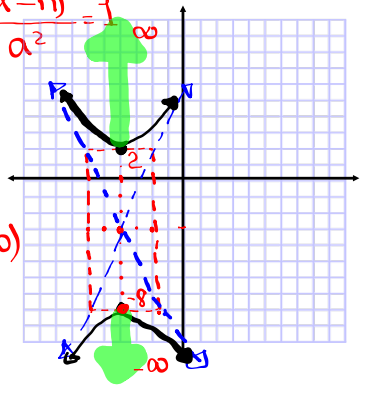
$a=2$

$b=5$

opens up & down

Domain: $(-\infty,\infty)$

Range: $(-\infty,-8]\cup[2,\infty)$



Ex:

$$16y^2 - 25(x+4)^2 = -400$$

Make RHS = 1 $\Leftrightarrow$ Divide everything by -400

$$\frac{16y^2}{-400} - \frac{25(x+4)^2}{-400} = \frac{-400}{-400}$$

$$-\frac{y^2}{25} + \frac{(x+4)^2}{16} = 1 \Rightarrow \frac{(x+4)^2}{16} - \frac{y^2}{25} = 1$$

opens right & left

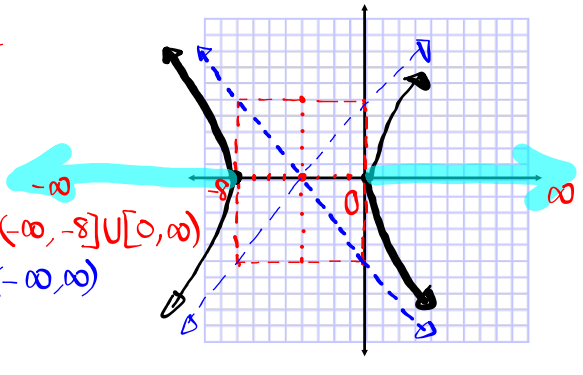
Center $(-4,0)$

$a = 4$

$b = 5$

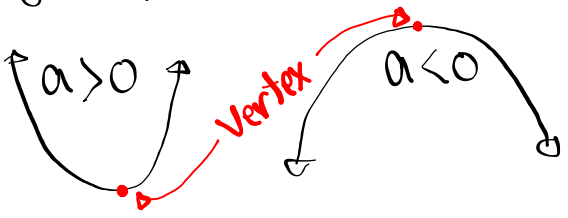
Domain: $(-\infty,-8]\cup[0,\infty)$

Range: $(-\infty,\infty)$



More on Parabola $a \neq 0$

$y = a(x-h)^2 + K$



$y = ax^2 + bx + c$

$h = \frac{-b}{2a}$

K = plug in h, and simplify

Vertex (h, K)

A.O.S. $x = h$

Y-Int $(0, ?)$

x-Int: $(?, 0)$

Ex: $y = 2(x-1)^2 + 3$

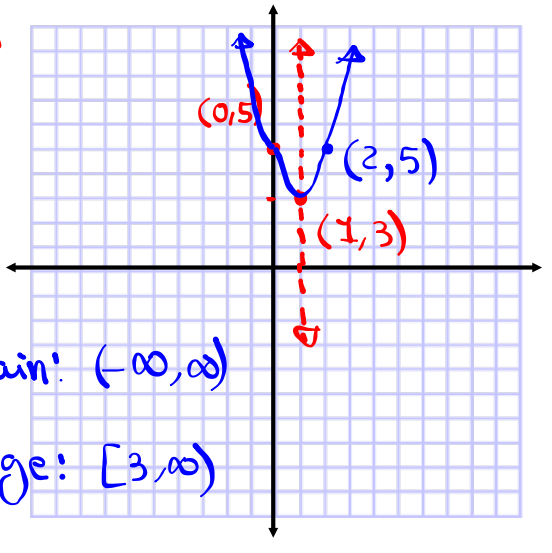
$y = a(x-h)^2 + K$

$a = 2$ → opens upward

$h = 1$, $K = 3$ } Vertex $(1, 3)$, A.O.S. $x = 1$

Y-Int $(0, 5)$

x-Int None



Domain: $(-\infty, \infty)$

Range: $[3, \infty)$

Ex: $y = -\frac{1}{2}(x+2)^2 - 4$

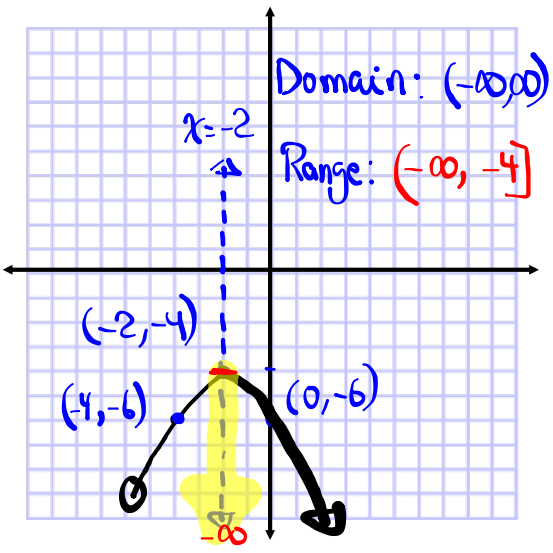
$y = a(x-h)^2 + k$

$a = -\frac{1}{2}$ opens downward

$h = -2$ Vertex $(-2,-4)$

$k = -4$ $\Rightarrow$ A.O.S. $x = -2$

Y-Int $(0,-6)$



Ex: $y = -x^2 - 8x$

$y = ax^2 + bx + c$

$a = -1$ opens downward

$b = -8$

$c = 0$

$h = \frac{-b}{2a} = \frac{-(-8)}{2(-1)} = \frac{8}{-2} = -4$

$k = -(-4)^2 - 8(-4) = -16 + 32 = 16$

$\Rightarrow$ Vertex $(-4,16)$

A.O.S. $x = -4$

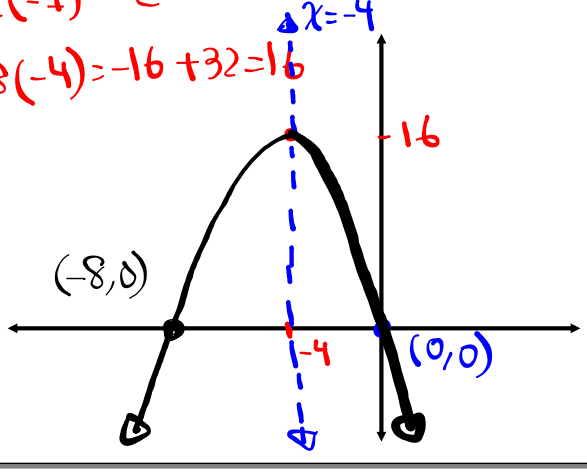
Y-Int $(0,0)$

Now graph

x-Ints $(-8,0), (0,0)$

Domain: $(-\infty,\infty)$

Range: $(-\infty,16]$



Ex: $y = x^2 - 2x - 8$

$y = ax^2 + bx + c$

$a = 1$ opens upward

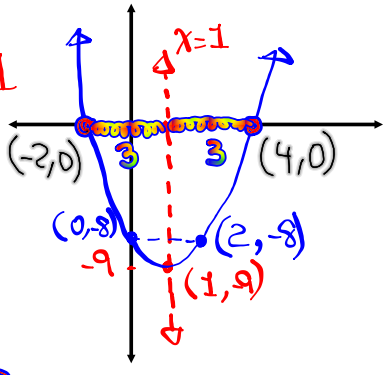
$b = -2$ $\quad h = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = \frac{2}{2} = 1$

$c = -8$ $\quad k = 1^2 - 2(1) - 8 = -9$

Vertex $(1, -9)$

A.O.S. $x = 1$

Y-Int $(0, -8)$



x-Int $\Rightarrow$ $y = 0$

$x^2 - 2x - 8 = 0$

$(x + 2)(x - 4) = 0$

$x = -2$ $\quad x = 4$

Solve $(3x - 5)^2 = -16$

Use S.R.M.

$3x - 5 = \pm\sqrt{-16}$

$3x - 5 = \pm 4i$

$3x = 5 \pm 4i$

$x = \frac{5}{3} \pm \frac{4}{3}i$

$\left\{\frac{5}{3} \pm \frac{4}{3}i\right\}$

Solve by Completing the Square method:

$x^2 + 10x + 7 = 0$

$-7 + 25 = 18$

$x^2 + 10x + 5^2 = -7 + 5^2$

$\frac{1}{2} \cdot 10 = 5$

$(x+5)^2 = 18$

Now S.R.M.

$x + 5 = \pm\sqrt{18}$

$x = -5 \pm \sqrt{9}\sqrt{2}$

$x = -5 \pm 3\sqrt{2}$

$\{-5 \pm 3\sqrt{2}\}$

Solve $3x^2 = 5x - 10$ by Quadratic Formula.

$3x^2 - 5x + 10 = 0$

$a = 3$

$b = -5$

$c = 10$

$b^2 - 4ac = (-5)^2 - 4(3)(10) = -95$

$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-(-5) \pm \sqrt{-95}}{2(3)}$

$= \frac{5 \pm \sqrt{95}\,i}{6}$

$= \frac{5}{6} \pm \frac{\sqrt{95}}{6}\,i$

$\left\{\frac{5}{6} \pm \frac{\sqrt{95}}{6}\,i\right\}$

Determine the type of Solutions for the equation $(2x+1)(x+3)=7$

Hint: Find discriminant, and discuss.

$2x^2+6x+x+3-7=0$

$2x^2+7x-4=0$

$a=2 \quad b=7 \quad c=-4$

$b^2-4ac=(7)^2-4(2)(-4)=81$

Since $b^2-4ac>0 \Rightarrow$ we get two real Solutions.

Find a quadratic equation in the form of $ax^2+bx+c=0$ with Solutions $-2\pm3i$.

$x=-2+3i \qquad x=-2-3i$

$x+2-3i=0 \qquad x+2+3i=0$

$(x+2-3i)(x+2+3i)=0$

Conjugates

$(x+2)^2-(3i)^2=0$

$(x+2)(x+2)-9i^2=0$

$x^2+2x+2x+4-9(-1)=0$

$x^2+4x+4+9=0 \rightarrow \boxed{x^2+4x+13=0}$

Solve $x^4 + 13x^2 + 36 = 0$

Let $u = x^2$

$u^2 = x^4$

$u^2 + 13u + 36 = 0$

$(u+9)(u+4) = 0$

$u = -9$ $\quad$ $u = -4$

$x^2 = -9$ $\quad$ $x^2 = -4$

$x = \pm\sqrt{-9}$ $\quad$ $x = \pm\sqrt{-4}$

$\boxed{x = \pm 3i}$ $\quad$ $\boxed{x = \pm 2i}$

$\{\pm 3i, \pm 2i\}$

Class QZ 21

Find a quadratic equation in $ax^2+bx+c=0$ form with solutions $3 \pm 2\sqrt{5}$.

$x = 3 + 2\sqrt{5}$ $\quad$ $x = 3 - 2\sqrt{5}$

$x - 3 - 2\sqrt{5} = 0$ $\quad$ $x - 3 + 2\sqrt{5} = 0$

$(x - 3 - 2\sqrt{5})(x - 3 + 2\sqrt{5}) = 0$

$(x-3)^2 - (2\sqrt{5})^2 = 0$

$(x-3)(x-3) - 4 \cdot 5 = 0$

$x^2 - 3x - 3x + 9 - 20 = 0$

$\boxed{x^2 - 6x - 11 = 0}$